International Mathematical Talent Search – Round 19

Problem 1/19. It is possible to replace each of the $\pm$ signs below by either $-$ or $+$ so that

$$\pm 1 \pm 2 \pm 3 \pm 4 \pm \cdots \pm 96 = 1996.$$

At most how many of the $\pm$ signs can be replaced by a $+$ sign?

Problem 2/19. We say (a, b, c) is a *primitive Heronian triple* if a, b, and c are positive integers with no common factors (other than 1), and if the area of the triangle whose sides measure a, b, and c is also an integer. Prove that if $a = 96$, then b and c must both be odd.

Problem 3/19. The numbers in the 7×8 rectangle shown on the right were obtained by putting together the 28 distinct dominoes of a standard set, recording the number of dots, ranging from 0 to 6 on each side of the dominoes, and then erasing the boundaries among them. Determine the original boundaries among the dominoes. (Note: Each domino consists of two adjoint squares, referred to as its sides.)

5	5	5	2	1	3	3	4
6	4	4	2	1	1	5	2
6	3	3	2	1	6	0	3
3	0	5	5	0	0	0	6
3	2	1	6	0	0	4	2
0	3	6	4	6	2	6	5
2	1	1	4	4	4	1	5

Problem 4/19. Suppose that f satisfies the functional equation

$$2f(x) + 3f\left(\frac{2x+29}{x-2}\right) = 100x + 80.$$

Find $f(3)$.

Problem 5/19. In the figure on the right, determine the area of the shaded octogon as a fraction of the area of the square, where the boundaries of the octogon are lines drawn from the vertices of the square to the midpoints of the opposite sides.