International Mathematical Talent Search – Round 3

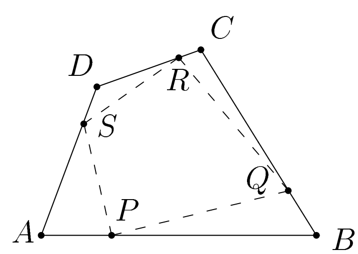
Problem 1/3. Note that if the product of any two distinct members of $\{1, 16, 27\}$ is increased by 9, the result is the perfect square of an integer. Find the unique positive integer n for which $n + 9$, $16n + 9$, and $27n + 9$ are also perfect squares.

Problem 2/3. Note that 1990 can be "turned into a square" by adding a digit on its right, and some digits on its left; i.e., $419904 = 648^2$. Prove that 1991 can not be turned into a square by the same procedure; i.e., there are no digits $d, x, y, \ldots$ such that $\ldots yx1991d$ is a perfect square.

Problem 3/3. Find k if P, Q, R, and S are points on the sides of quadrilateral $ABCD$ so that

$$\frac{AP}{PB} = \frac{BQ}{QC} = \frac{CR}{RD} = \frac{DS}{SA} = k,$$

and the area of quadrilateral $PQRS$ is exactly 52% of the area of quadrilateral $ABCD$.



Problem 4/3. Let n points with integer coordinates be given in the xy-plane. What is the minimum value of n which will ensure that three of the points are the vertices of a triangle with integer (possibly, 0) area?

Problem 5/3. Two people, A and B, play the following game with a deck of 32 cards. With A starting, and thereafter the players alternating, each player takes either 1 card or a prime number of cards. Eventually all of the cards are chosen, and the person who has none to pick up is the loser. Who will win the game if they both follow optimal strategy?