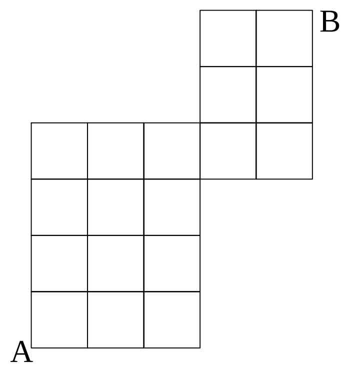
International Mathematical Talent Search – Round 34

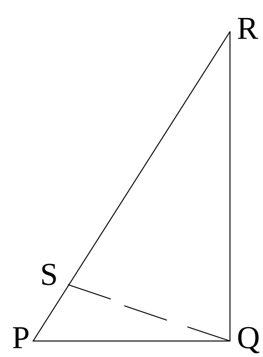
Problem 1/34. The number N consists of 1999 digits such that if each pair of consecutive digits in N were viewed as a two-digit number, then that number would either be a multiple of 17 or a multiple of 23. The sum of the digits of N is 9599. Determine the rightmost ten digits of N.

Problem 2/34. Let $\mathcal{C}$ be the set of non-negative integers which can be expressed as $1999s + 2000t$ where s and t are also non-negative integers.
(a) Show that $3,994,001$ is not in $\mathcal{C}$.
(b) Show that if $0 \leq n \leq 3,994,001$ and n is an integer not in $\mathcal{C}$, then $3,994,001 - n$ is in $\mathcal{C}$.

Problem 3/34. The figure on the right shows the map of Squareville, where each city block is of the same length. Two friends, Alexandra and Brianna, live at corners marked by A and B, respectively. They start walking toward each other's house, leaving at the same time, walking with the same speed, and independently choosing a path to the other's house with uniform distribution out of all possible minimum-distance paths (that is, all minimum-distance paths are equally likely). What is the probability that they will meet?



Problem 4/34. In $\triangle PQR$, $PQ = 8$, $QR = 13$, and $RP = 15$. Prove that there is a point S on the line segment $\overline{PR}$, but not at its endpoints, such that PS and QS are also integers.



Problem 5/34. In $\triangle ABC$, $AC > BC$, CM is the median, and CH is the altitude emanating from C, as shown in the figure on the right. Determine the measure of $\angle MCH$, if $\angle ACM$ and $\angle BCH$ each have measure 17°.