International Mathematical Talent Search – Round 39

Problem 1/39. Find the smallest positive integer with the property that it has divisors ending in every decimal digit; i.e., divisors ending in $0, 1, 2, \ldots, 9$.

Problem 2/39. Assume that the irreducible fractions between 0 and 1, with denominators at most 99, are listed in ascending order. Determine which two fractions are adjacent to $\frac{17}{76}$ in this listing.

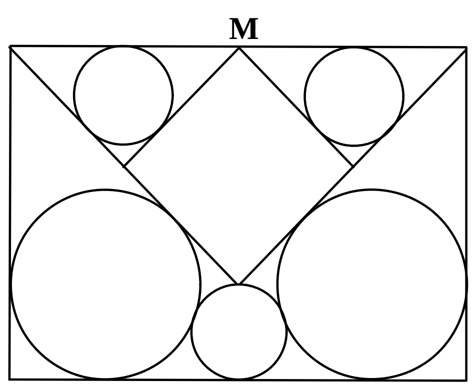
Problem 3/39. Let $p(x) = x^5 + x^2 + 1$ have roots r_1, r_2, r_3, r_4, r_5. Let $q(x) = x^2 - 2$. Determine the product $q(r_1)q(r_2)q(r_3)q(r_4)q(r_5)$.

Problem 4/39. Assume that each member of the sequence $< \diamond_i >_{i=1}^{\infty}$ is either a $+$ or $-$ sign. Determine the appropriate sequence of $+$ and $-$ signs so that

$$2 = \sqrt{6 \diamond_1 \sqrt{6 \diamond_2 \sqrt{6 \diamond_3 \cdots}}}.$$

Also determine what sequence of signs is necessary if the sixes in the nested roots are replaced by sevens. List all integers that work in the place of the sixes and the sequence of signs that are needed with them.

Problem 5/39. Three isosceles right triangles are erected from the larger side of a rectangle into the interior of the rectangle, as shown on the right, where M is the midpoint of that side. Five circles are inscribed tangent to some of the sides and to one another as shown. One of the circles touches the vertex of the largest triangle.



Find the ratios among the radii of the five circles.