International Mathematical Talent Search – Round 41

Problem 1/41. Determine the unique positive two-digit integers m and n for which the approximation $\dfrac{m}{n} = .2328767$ is accurate to the seven decimals; i.e., $0.2328767 \leq m/n < 0.2328768$.

Problem 2/41. It is well known that there are infinitely many triples of integers (a, b, c) whose greatest common divisor is 1 and which satisfy the equation $a^2 + b^2 = c^2$.

Prove that there are also infinitely many triples of integers (r, s, t) whose greatest common divisor is 1 and which satisfy the equation $(rs)^2 + (st)^2 = (tr)^2$.

Problem 3/41. Suppose $\dfrac{\cos 3x}{\cos x} = \dfrac{1}{3}$ for some angle x, $0 \leq x \leq \dfrac{\pi}{2}$. Determine $\dfrac{\sin 3x}{\sin x}$ for the same x.

Problem 4/41. The projective plane of order three consists of 13 points and 13 lines. These lines are not Euclidean straight lines; instead they are sets of four points with the properties that each pair of lines has exactly one point in common and each pair of points has exactly one line that contains both points. Suppose the points are labeled 1 through 13 and six of the lines are $A = \{1, 2, 4, 8\}$, $B = \{1, 3, 5, 9\}$, $C = \{2, 3, 6, 10\}$, $D = \{4, 5, 10, 11\}$, $E = \{4, 6, 9, 12\}$, and $F = \{5, 6, 8, 13\}$. What is the line that contains 7 and 8?

Problem 5/41. In $\triangle PQR$, $QR < PR < PQ$ so that the exterior angle bisector through P intersects ray $\overrightarrow{QR}$ at point S, and the exterior angle bisector at R intersects ray $\overrightarrow{PQ}$ at point T, as shown on the right. Given that $PR = PS = RT$, determine, with proof, the measure of $\angle PRQ$.