Canadian Mathematical Olympiad
1992

Problem 1

Prove that the product of the first n natural numbers is divisible by the sum of the first n natural numbers if and only if $n+1$ is not an odd prime.

Problem 2

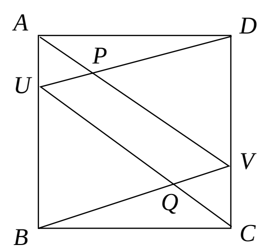
For $x, y, z \geq 0$, establish the inequality

$$x(x-z)^2 + y(y-z)^2 \geq (x-z)(y-z)(x+y-z)$$

and determine when equality holds.

Problem 3

In the diagram, $ABCD$ is a square, with U and V interior points of the sides AB and CD respectively. Determine all the possible ways of selecting U and V so as to maximize the area of the quadrilateral $PUQV$.



Problem 4

Solve the equation

$$x^2 + \frac{x^2}{(x+1)^2} = 3.$$

Problem 5

A deck of $2n+1$ cards consists of a joker and, for each number between 1 and n inclusive, two cards marked with that number. The $2n+1$ cards are placed in a row, with the joker in the middle. For each k with $1 \leq k \leq n$, the two cards numbered k have exactly $k-1$ cards between them. Determine all the values of n not exceeding 10 for which this arrangement is possible. For which values of n is it impossible?