

STUDENT:

Reference Copy

SCHOOL:

Canada Way H.S., Ottawa

EXAM ID: **10101**

2018 Canadian Mathematical Olympiad

Official Exam Booklet



Student's Instructions

THE ONLY INSTRUMENTS PERMITTED ARE PENCILS, PENS, ERASERS, WHITEOUT, RULER, AND COMPASSES. NO OTHER AIDS ARE PERMITTED.

1. Please verify that your name and school at the top of this page is correct..
2. You cannot ask any questions concerning the examination. If you unsure of a problem make a note and then state and solve what you consider to be a valid non-trivial interpretation of the problem.
3. This exam consists of 5 questions. The questions are all of equal value. Partial marks may be awarded. Each solution must be justified and should be clear, concise and complete since good presentation counts. Special awards and/or extra credit may be given for elegant solutions and/or valid generalizations of particular problems.
4. All solutions must be written in this book, written in pen or pencil on one side of paper only. The backs of pages should be reserved for rough work. You may include extra pages if there is not enough room to answer a question. Please ensure extra pages include the question number and your exam number. Do not include your name.
5. At the end of the examination, the proctor (exam supervisor) must sign the declaration below and forward this booklet for marking. You should remove the back page of this booklet and keep it – it contains your unique Exam ID you will need to look up your results later.
6. Please do not discuss the contents of the exam online for the next 24 hours.

Proctor's Declaration

I confirmed the identity of the student whose name is written above and ensured that the student wrote this examination without assistance from any sources whatsoever on March 28th, 2018 during the official time period assigned. I declare I am not family-related to this student.

Name of Proctor (printed)

Signature

Signed Declaration of Proctor is Required to Validate the Exam

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DO NOT WRITE ON THIS PAGE

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3



4



5



TOTAL



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1. Consider an arrangement of tokens in the plane, not necessarily at distinct points. We are allowed to apply a sequence of moves of the following kind: Select a pair of tokens at points A and B and move both of them to the midpoint of A and B .

We say that an arrangement of n tokens is *collapsible* if it is possible to end up with all n tokens at the same point after a finite number of moves. Prove that every arrangement of n tokens is collapsible if and only if n is a power of 2.





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2. Let five points on a circle be labelled A , B , C , D , and E in clockwise order. Assume $AE = DE$ and let P be the intersection of AC and BD . Let Q be the point on the line through A and B such that A is between B and Q and $AQ = DP$. Similarly, let R be the point on the line through C and D such that D is between C and R and $DR = AP$. Prove that PE is perpendicular to QR .





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3. Two positive integers a and b are *prime-related* if $a = pb$ or $b = pa$ for some prime p . Find all positive integers n , such that n has at least three divisors, and all the divisors can be arranged without repetition in a circle so that any two adjacent divisors are prime-related.

Note that 1 and n are included as divisors.





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4. Find all polynomials $p(x)$ with real coefficients that have the following property: There exists a polynomial $q(x)$ with real coefficients such that

$$p(1) + p(2) + p(3) + \cdots + p(n) = p(n)q(n)$$

for all positive integers n .





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5. Let k be a given even positive integer. Sarah first picks a positive integer N greater than 1 and proceeds to alter it as follows: every minute, she chooses a prime divisor p of the current value of N , and multiplies the current N by $p^k - p^{-1}$ to produce the next value of N . Prove that there are infinitely many even positive integers k such that, no matter what choices Sarah makes, her number N will at some point be divisible by 2018.





Canada Way H.S., Ottawa

Thank you, Reference Copy, for participating in the Canadian Mathematical Olympiad this year. **Your exam number is 10101.** You will need this number in order to view your CMO score online. We will email you at studentemail@cms.math.ca when the scores are available.

The CMO scores will be posted at: <https://cmo.math.ca/>

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