

## International Mathematical Talent Search – Round 2

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**Problem 1/2.** What is the smallest integer multiple of 9997, other than 9997 itself, which contains only odd digits?

**Problem 2/2.** Show that every triangle can be dissected into nine convex nondegenerate pentagons.

**Problem 3/2.** Prove that if  $x, y,$  and  $z$  are pairwise relatively prime positive integers, and if  $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ , then  $x + y, x - z,$  and  $y - z$  are perfect squares of integers.

**Problem 4/2.** Let  $a, b, c,$  and  $d$  be the areas of the triangular faces of a tetrahedron, and let  $h_a, h_b, h_c,$  and  $h_d$  be the corresponding altitudes of the tetrahedron. If  $V$  denotes the volume of the tetrahedron, prove that

$$(a + b + c + d)(h_a + h_b + h_c + h_d) \geq 48V.$$

**Problem 5/2.** Prove that there are infinitely many positive integers  $n$  such that the  $n \times n \times n$  box can not be filled completely with  $2 \times 2 \times 2$  and  $3 \times 3 \times 3$  solid cubes.