

## International Mathematical Talent Search – Round 24

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**Problem 1/24.** The lattice points of the first quadrant are numbered as shown in the diagram on the right. Thus, for example, the 19th lattice point is  $(2, 3)$ , while the 97th lattice point is  $(8, 5)$ . Determine, with proof, the 1997th lattice point in this scheme.

	○	○	○	○	○	○	○	○
22	35	○	○	○	○	○	○	○
21	23	34	○	○	○	○	○	○
11	20	24	33	○	○	○	○	○
10	12	19	25	32	○	○	○	○
4	9	13	18	26	31	○	○	○
3	5	8	14	17	27	30	○	○
1	2	6	7	15	16	28	29	○

**Problem 2/24.** Let  $N_k = 131313 \dots 131$  be the  $(2k + 1)$ -digit number (in base 10), formed from  $k + 1$  copies of 1 and  $k$  copies of 3. Prove that  $N_k$  is not divisible by 31 for any value of  $k = 1, 2, 3, \dots$

**Problem 3/24.** In  $\triangle ABC$ , let  $AB = 52$ ,  $BC = 64$ ,  $CA = 70$ , and assume that  $P$  and  $Q$  are points chosen on sides  $AB$  and  $AC$ , respectively, so that  $\triangle APQ$  and quadrilateral  $PBCQ$  have the same area and the same perimeter. Determine the square of the length of the segment  $PQ$ .

**Problem 4/24.** Determine the positive integers  $x < y < z$  for which

$$\frac{1}{x} - \frac{1}{xy} - \frac{1}{xyz} = \frac{19}{97}.$$

**Problem 5/24.** Let  $P$  be a convex planar polygon with  $n$  vertices, and from each vertex of  $P$  construct perpendiculars to the  $n - 2$  sides (or extensions thereof) of  $P$  not meeting at that vertex. Prove that either one of these perpendiculars is completely in the interior of  $P$  or it is a side of  $P$ .