International Mathematical Talent Search – Round 29

Problem 1/29.Several pairs of positive integers (m, n) satisfy the equation $19m + 90 + 8n = 1998$. Of these, $(100, 1)$ is the pair with the smallest value for n. Find the pair with the smallest value for m.

Problem 2/29.Determine the smallest rational number $\frac{r}{s}$ such that $\frac{1}{k} + \frac{1}{m} + \frac{1}{n} \leq \frac{r}{s}$ whenever $k, m,$ and n are positive integers that satisfy the inequality $\frac{1}{k} + \frac{1}{m} + \frac{1}{n} < 1$.

Problem 3/29.It is possible to arrange eight of the nine numbers

$$2, 3, 4, 7, 10, 11, 12, 13, 15$$

in the vacant squares of the 3 by 4 array shown on the right so that the arithmetic average of the numbers in each row and in each column is the same integer. Exhibit such an arrangement, and specify which one of the nine numbers must be left out when completing the array.

1			
	9		5
		14	

Problem 4/29.Show that it is possible to arrange seven distinct points in the plane so that among any three of these seven points, two of the three points are a unit distance apart. (Your solution should include a carefully prepared sketch of the seven points, along with all segments that are of unit length.)

Problem 5/29.The figure on the right shows the ellipse

$$\frac{(x-19)^2}{19} + \frac{(y-98)^2}{98} = 1998.$$

Let $R_1, R_2, R_3,$ and R_4 denote those areas within the ellipse that are in the 1st, 2nd, 3rd, and 4th quadrants, respectively. Determine the value of

$$R_1 - R_2 + R_3 - R_4.$$