PROBLEMS FOR APRIL

Solutions should be submitted to
Dr. Valeria Pandelieva
641 Kirkwood Avenue
Ottawa, ON K1Z 5X5
no later than **May 31, 2001**, and no sooner than *May 21, 2001*.

73. Solve the equation:

$$\left(\sqrt{2+\sqrt{2}}\right)^x + \left(\sqrt{2-\sqrt{2}}\right)^x = 2^x \ .$$

74. Prove that among any group of $n+2$ natural numbers, there can be found two numbers so that their sum or their difference is divisible by $2n$.

75. Three consecutive natural numbers, larger than 3, represent the lengths of the sides of a triangle. The area of the triangle is also a natural number.

(a) Prove that one of the altitudes "cuts" the triangle into two triangles, whose side lengths are natural numbers.

(b) The altitude identified in (a) divides the side which is perpendicular to it into two segments. Find the difference between the lengths of these segments.

76. Solve the system of equations:

$$\log x + \frac{\log(xy^8)}{\log^2 x + \log^2 y} = 2 \ ,$$

$$\log y + \frac{\log(x^8/y)}{\log^2 x + \log^2 y} = 0 \ .$$

(The logarithms are taken to base 10.)

77. n points are chosen from the circumference or the interior of a regular hexagon with sides of unit length, so that the distance between any two of them is less than $\sqrt{2}$. What is the largest natural number n for which this is possible?

78. A truck travelled from town A to town B over several days. During the first day, it covered $1/n$ of the total distance, where n is a natural number. During the second day, it travelled $1/m$ of the remaining distance, where m is a natural number. During the third day, it travelled $1/n$ of the distance remaining after the second day, and during the fourth day, $1/m$ of the distance remaining after the third day. Find the values of m and n if it is known that, by the end of the fourth day, the truck had travelled $3/4$ of the distance between A and B. (Without loss of generality, assume that $m < n$.)