PROBLEMS FOR JULY

Please send your solutions to
Dr. Valeria Pandelieva
641 Kirkwood Avenue
Ottawa, ON K1Z 5X5
no later than **August 31, 2001** and no sooner than *August 15, 2001*.

Note. There was an unfortunate error in the statement of Problem 77. I would like to apologize to students who tried to solve the problem and did not get the point of it because of the mistake. A corrected version is listed below, and solutions can be mailed to Dr. Pandelieva. Some of the original solvers detected the error and sent solutions to the problem that was intended; such students need not send anything further on this problem. (If the statement of a problem on a competition seems fishy, draw attention to what you think may be the probably error, explicitly state a *nontrivial* formulation of the problem and solve that.) (*E. Barbeau*)

77. n points are chosen from the circumference or the interior of a regular hexagon with sides of unit length, so that the distance between any two of them is **not** less that $\sqrt{2}$. What is the largest natural number n for which this is possible?

91. A square and a regular pentagon are inscribed in a circle. The nine vertices are all distinct and divide the circumference into nine arcs. Prove that at least one of them does not exceed $1/40$ of the circumference of the circle.

92. Consider the sequence 200125, 2000125, 20000125, $\cdots$, $200\cdots00125$, $\cdots$ (in which the nth number has $n+1$ digits equal to zero). Can any of these numbers be the square or the cube of an integer?

93. For any natural number n, prove the following inequalities:

$$2^{(n-1)/(2^{n-2})} \leq \sqrt{2}\sqrt[4]{4}\sqrt[8]{8}\cdots\sqrt[2^n]{2^n} < 4 \ .$$

94. ABC is a right triangle with arms a and b and hypotenuse $c = |AB|$; the area of the triangle is s square units and its perimeter is $2p$ units. The numbers a, b and c are positive integers. Prove that s and p are also positive integers and that s is a multiple of p.

95. The triangle ABC is isosceles with equal sides AC and BC. Two of its angles measure $40°$. The interior point M is such that $\angle MAB = 10°$ and $\angle MBA = 20°$. Determine the measure of $\angle CMB$.

96. Find all prime numbers p for which all three of the numbers $p^2 - 2$, $2p^2 - 1$ and $3p^2 + 4$ are also prime.