PROBLEMS FOR DECEMBER

Please send your solution to
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no later than January 15, 2003. It is important that your complete mailing address and your email address appear on the front page.

Notes. An *isosceles* tetrahedron is one for which the three pairs of oppposite edges are equal. For integers a, b and n, $a \equiv b$, modulo n, iff $a - b$ is a multiple of n.

192. Let ABC be a triangle, D be the midpoint of AB and E a point on the side AC for which $AE = 2EC$. Prove that BE bisects the segment CD.

193. Determine the volume of an isosceles tetrahedron for which the pairs of opposite edges have lengths a, b, c. Check your answer independently for a regular tetrahedron.

194. Let ABC be a triangle with incentre I. Let M be the midpoint of BC, U be the intersection of AI produced with BC, D be the foot of the perpendicular from I to BC and P be the foot of the perpendicular from A to BC. Prove that

$$|PD||DM| = |DU||PM| \ .$$

195. Let $ABCD$ be a convex quadrilateral and let the midpoints of AC and BD be P and Q respectively, Prove that

$$|AB|^2 + |BC|^2 + |CD|^2 + |DA|^2 = |AC|^2 + |BD|^2 + 4|PQ|^2 \ .$$

196. Determine five values of p for which the polynomial $x^2 + 2002x - 1002p$ has integer roots.

197. Determine all integers x and y that satisfy the equation $x^3 + 9xy + 127 = y^3$.

198. Let p be a prime number and let $f(x)$ be a polynomial of degree d with integer coefficients such that $f(0) = 0$ and $f(1) = 1$ and that, for every positive integer n, $f(n) \equiv 0$ or $f(n) \equiv 1$, modulo p. Prove that $d \geq p - 1$. Give an example of such a polynomial.