Problems for August, 2005

Please send your solution to
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no later than September 15, 2005 It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

395. None of the nine participants at a meeting speaks more than three languages. Two of any three speakers speak a common language. Show that there is a language spoken by at least three participants.

396. Place 32 white and 32 black checkers on a 8×8 square chessboard. Two checkers of different colours form a *related pair* if they are placed in either the same row or the same column. Determine the maximum and the minimum number of related pairs over all possible arrangements of the 64 checkers.

397. The altitude from A of triangle ABC intersects BC in D. A circle touches BC at D, intersectes AB at M and N, and intersects AC at P and Q. Prove that

$$(AM + AN) : AC = (AP + AQ) : AB \ .$$

398. Given three disjoint circles in the plane, construct a point in the plane so that all three circles subtend the same angle at that point.

399. Let n and k be positive integers for which $k < n$. Determine the number of ways of choosing k numbers from $\{1, 2, \cdots, n\}$ so that no three consecutive numbers appear in any choice.

400. Let a_r and b_r $(1 \leq r \leq n)$ be real numbers for which $a_1 \geq a_2 \geq \cdots \geq a_n \geq 0$ and

$$b_1 \geq a_1 \ , \quad b_1 b_2 \geq a_1 a_2 \ , \quad b_1 b_2 b_3 \geq a_1 a_2 a_3 \ , \quad \cdots \quad , b_1 b_2 \cdots b_n \geq a_1 a_2 \cdots a_n \ .$$

Show that

$$b_1 + b_2 + \cdots + b_n \geq a_1 + a_2 + \cdots + a_n \ .$$

401. Find integers are arranged in a circle. The sum of the five integers is positive, but at least one of them is negative. The configuration is changed by the following moves: at any stage, a negative integer is selected and its sign is changed; this negative integer is added to each of its neighbours (*i.e.*, its absolute value is subtracted from each of its neighbours).

Prove that, regardless of the negative number selected for each move, the process will eventually terminate with all integers nonnegative in exactly the same number of moves with exactly the same configuration.