

## PROBLEMS FOR DECEMBER

Please send your solution to

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no later than January 31, 2006. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

*Note:* The function  $f$  defined on the real numbers and taking real values is *increasing* if and only if, for  $x < y$ ,  $f(x) \leq f(y)$ .

472. Find all integers  $x$  for which

$$(4-x)^{4-x} + (5-x)^{5-x} + 10 = 4^x + 5^x .$$

473. Let  $ABCD$  be a quadrilateral; let  $M$  and  $N$  be the respective midpoint of  $AB$  and  $BC$ ; let  $P$  be the point of intersection of  $AN$  and  $BD$ , and  $Q$  be the point of intersection of  $DM$  and  $AC$ . Suppose the  $3BP = BD$  and  $3AQ = AC$ . Prove that  $ABCD$  is a parallelogram.

474. Solve the equation for positive real  $x$ :

$$(2^{\log_5 x} + 3)^{\log_5 2} = x - 3 .$$

475. Let  $z_1, z_2, z_3, z_4$  be distinct complex numbers for which  $|z_1| = |z_2| = |z_3| = |z_4|$ . Suppose that there is a real number  $t \neq 1$  for which

$$|tz_1 + z_2 + z_3 + z_4| = |z_1 + tz_2 + z_3 + z_4| = |z_1 + z_2 + tz_3 + z_4| .$$

Show that, in the complex plane,  $z_1, z_2, z_3, z_4$  lie at the vertices of a rectangle.

476. Let  $p$  be a positive real number and let  $|x_0| \leq 2p$ . For  $n \geq 1$ , define

$$x_n = 3x_{n-1} - \frac{1}{p^2}x_{n-1}^3 .$$

Determine  $x_n$  as a function of  $n$  and  $x_0$ .

477. Let  $S$  consist of all real numbers of the form  $a + b\sqrt{2}$ , where  $a$  and  $b$  are integers. Find all functions that map  $S$  into the set  $\mathbf{R}$  of reals such that (1)  $f$  is increasing, and (2)  $f(x+y) = f(x) + f(y)$  for all  $x, y$  in  $S$ .

478. Solve the equation

$$\sqrt{2 + \sqrt{2 + \sqrt{2 + x}}} + \sqrt{3}\sqrt{2 - \sqrt{2 + \sqrt{2 + x}}} = 2x$$

for  $x \geq 0$