PROBLEMS FOR MAY

Please send your solution to

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no later than August 31, 2008. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

549. The set E consists of 37 two-digit natural numbers, none of them a multiple of 10. Prove that, among the elements of E, we can find at least five numbers, such that any two of them have different tens digits and different units digits.

550. The functions $f(x)$ and $g(x)$ are defined by the equations: $f(x) = 2x^2 + 2x - 4$ and $g(x) = x^2 - x + 2$.

(a) Find all real numbers x for which $f(x)/g(x)$ is a natural number.

(b) Find the solutions of the inequality

$$\sqrt{f(x)} + \sqrt{g(x)} \geq 2 .$$

551. The numbers 1, 2, 3 and 4 are written on the circumference of a circle, in this order. Alice and Bob play the following game: On each turn, Alice adds 1 to two adjacent numbers, while Bob switches the places of two adjacent numbers. Alice wins the game, if after her turn, all numbers on the circle are equal. Does Bob have a strategy to prevent Alice from winning the game? Justify your answer.

552. Two real nonnegative numbers a and b satisfy the inequality $ab \geq a^3 + b^3$. Prove that $a + b \leq 1$.

553. The convex quadrilateral $ABCD$ is concyclic with side lengths $|AB| = 4$, $|BC| = 3$, $|CD| = 2$ and $|DA| = 1$. What is the length of the radius of the circumcircle of $ABCD$? Provide an exact value of the answer.

554. Determine all real pairs (x, y) that satisfy the system of equations:

$$3\sqrt[3]{x^2y^5} = 4(y^2 - x^2)$$

$$5\sqrt[3]{x^4y} = y^2 + x^2 .$$

555. Let ABC be a triangle, all of whose angles do not exceed $90°$. The points K on side AB, M on side AC and N on side BC are such that $KM \perp AC$ and $KN \perp BC$. Prove that the area $[ABC]$ of triangle ABC is at least 4 times as great as the area $[KMN]$ of triangle KMN, *i.e.*, $[ABC] \geq 4[KMN]$. When does equality hold?