PROBLEMS FOR AUGUST

Solutions should be submitted to
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Solution to these problems should be postmarked no later than **September 30, 2000**.

Note: For any real number x, $\lfloor x \rfloor$ (the *floor* of x) is equal to the greatest integer that is less than or equal to x.

25. Let a, b, c be non-negative numbers such that $a+b+c=1$. Prove that

$$\frac{ab}{c+1}+\frac{bc}{a+1}+\frac{ca}{b+1}\le\frac{1}{4}\quad .$$

When does equality hold?

26. Each of m cards is labelled by one of the numbers $1, 2, \cdots, m$. Prove that, if the sum of labels of any subset of cards is not a multiple of $m+1$, then each card is labelled by the same number.

27. Find the least number of the form $|36^m - 5^n|$ where m and n are positive integers.

28. Let A be a finite set of real numbers which contains at least two elements and let $f : A \longrightarrow A$ be a function such that $|f(x)-f(y)| < |x-y|$ for every $x, y \in A$, $x \neq y$. Prove that there is $a \in A$ for which $f(a)=a$. Does the result remain valid if A is not a finite set?

29. Let A be a nonempty set of positive integers such that if $a \in A$, then $4a$ and $\lfloor \sqrt{a} \rfloor$ both belong to A. Prove that A is the set of all positive integers.

30. Find a point M within a regular pentagon for which the sum of its distances to the vertices is minimum.