PROBLEMS FOR NOVEMBER

Solutions should be submitted to
Dr. Dragos Hrimiuk
Department of Mathematics
University of Alberta
Edmonton, AB T6G 2G1
no later than **December 31, 2000**.

43. Two players pay a game: the first player thinkgs of n integers x_1, x_2, $\cdots$, x_n, each with one digit, and the second player selects some numbers a_1, a_2, $\cdots$, a_n and asks what is the vlaue of the sum $a_1x_1 + a_2x_2 + \cdots + a_nx_n$. What is the minimum number of questions used by the second player to find the integers a_1, x_2, $\cdots$, x_n?

44. Find the permutation $\{a_1, a_2, \cdots, a_n\}$ of the set $\{1, 2, \cdots, n\}$ for which the sum

$$S = |a_2 - a_1| + |a_3 - a_2| + \cdots + |a_n - a_{n-1}|$$

has maximum value.

45. Prove that there is no polynomial $p(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_0$ with integer coefficients a_i for which $p(m)$ is a prime number for every integer m.

46. Let $a_1 = 2$, $a_{n+1} = \frac{a_n+2}{1-2a_n}$ for $n = 1, 2, \cdots$. Prove that

(a) $a_n \neq 0$ for each positive integer n;

(b) there is no integer $p \geq 1$ for which $a_{n+p} = a_n$ for every integer $n \geq 1$ (*i.e.*, the sequence is not periodic).

47. Let $a_1, a_2, \cdots, a_n$ be positive real numbers such that $a_1a_2 \cdots a_n = 1$. Prove that

$$\sum_{k=1}^{n} \frac{1}{s - a_k} \leq 1$$

where $s = 1 + a_1 + a_2 + \cdots + a_n$.

48. Let $A_1A_2 \cdots A_n$ be a regular $n-$gon and d an arbitrary line. The parallels through A_i to d intersect its circumcircle respectively at B_i $(i = 1, 2, \cdots, n$. Prove that the sum

$$S = |A_1B_1|^2 + \cdots + |A_nB_n|^2$$

is independent of d.