PROBLEMS FOR FEBRUARY

Please send your solutions to
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no later than March 15, 2002.

127. Let

$$A = 2^n + 3^n + 216(2^{n-6} + 3^{n-6})$$

and

$$B = 4^n + 5^n + 8000(4^{n-6} + 5^{n-6})$$

where $n > 6$ is a natural number. Prove that the fraction A/B is reducible.

128. Let n be a positive integer. On a circle, n points are marked. The number 1 is assigned to one of them and 0 is assigned to the others. The following operation is allowed: Choose a point to which 1 is assigned and then assign $(1 - a)$ and $(1 - b)$ to the two adjacent points, where a and b are, respectively, the numbers assigned to these points before. Is it possible to assign 1 to all points by applying this operation several times if (a) $n = 2001$ and (b) $n = 2002$?

129. For every integer n, a nonnegative integer $f(n)$ is assigned such that

(a) $f(mn) = f(m) + f(n)$ for each pair m, n of natural numbers;

(b) $f(n) = 0$ when the rightmost digit in the decimal representation of the number n is 3; and

(c) $f(10) = 0$.

Prove that $f(n) = 0$ for any natural number n.

130. Let $ABCD$ be a rectangle for which the respective lengths of AB and BC are a and b. Another rectangle is circumscribed around $ABCD$ so that each of its sides passes through one of the vertices of $ABCD$. Consider all such rectangles and, among them, find the one with a maximum area. Express this area in terms of a and b.

131. At a recent winter meeting of the Canadian Mathematical Society, some of the attending mathematicians were friends. It appeared that every two mathematicians, that had the same number of friends among the participants, did not have a common friend. Prove that there was a mathematician who had only one friend.

132. Simplify the expression

$$\sqrt[5]{3\sqrt{2} - 2\sqrt{5}} \cdot \sqrt[10]{\frac{6\sqrt{10} + 19}{2}} .$$