PROBLEMS FOR MAY

145. Let ABC be a right triangle with $\angle A = 90^\circ$. Let P be a point on the hypotenuse BC, and let Q and R be the respective feet of the perpendiculars from P to AC and AB. For what position of P is the length of QR minimum?

146. Suppose that ABC is an equilateral triangle. Let P and Q be the respective midpoint of AB and AC, and let U and V be points on the side BC with $4BU = 4VC = BC$ and $2UV = BC$. Suppose that PV are joined and that W is the foot of the perpendicular from U to PV and that Z is the foot of the perpendicular from Q to PV.

    Explain how that four polygons $APZQ$, $BUWP$, $CQZV$ and UVW can be rearranged to form a rectangle. Is this rectangle a square?

147. Let $a > 0$ and let n be a positive integer. Determine the maximum value of

$$\frac{x_1 x_2 \cdots x_n}{(1 + x_1)(x_1 + x_2) \cdots (x_{n-1} + x_n)(x_n + a^{n+1})}$$

    subject to the constraint that $x_1, x_2, \cdots, x_n > 0$.

148. For a given prime number p, find the number of distinct sequences of natural numbers (positive integers) $\{a_0, a_1, \cdots, a_n \cdots\}$ satisfying, for each positive integer n, the equation

$$\frac{a_0}{a_1} + \frac{a_0}{a_2} + \cdots + \frac{a_0}{a_n} + \frac{p}{a_{n+1}} = 1 \ .$$

149. Consider a cube concentric with a parallelepiped (rectangular box) with sides $a < b < c$ and faces parallel to that of the cube. Find the side length of the cube for which the difference between the volume of the union and the volume of the intersection of the cube and parallelepiped is minimum.

150. The area of the bases of a truncated pyramid are equal to S_1 and S_2 and the total area of the lateral surface is S. Prove that, if there is a plane parallel to each of the bases that partitions the truncated pyramid into two truncated pyramids within each of which a sphere can be inscribed, then

$$S = (\sqrt{S_1} + \sqrt{S_2})(\sqrt[4]{S_1} + \sqrt[4]{S_2})^2 \ .$$