PROBLEMS FOR JULY

Please send your solution to
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no later than September 20, 2004. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

318. Solve for integers x, y, z the system

$$1 = x + y + z = x^3 + y^3 + z^2 .$$

[Note that the exponent of z on the right is 2, not 3.]

319. Suppose that a, b, c, x are real numbers for which $abc \neq 0$ and

$$\frac{xb + (1-x)c}{a} = \frac{xc + (1-x)a}{b} = \frac{xa + (1-x)b}{c} .$$

Prove that $a = b = c$.

320. Let L and M be the respective intersections of the internal and external angle bisectors of the triangle ABC at C and the side AB produced. Suppose that $CL = CM$ and that R is the circumradius of triangle ABC. Prove that

$$|AC|^2 + |BC|^2 = 4R^2 .$$

321. Determine all positive integers k for which $k^{1/(k-7)}$ is an integer.

322. The real numbers u and v satisfy

$$u^3 - 3u^2 + 5u - 17 = 0$$

and

$$v^3 - 3v^2 + 5v + 11 = 0 .$$

Determine $u + v$.

323. Alfred, Bertha and Cedric are going from their home to the country fair, a distance of 62 km. They have a motorcycle with sidecar that together accommodates at most 2 people and that can travel at a maximum speed of 50 km/hr. Each can walk at a maximum speed of 5 km/hr. Is it possible for all three to cover the 62 km distance within 3 hours?

324. The base of a pyramid $ABCDV$ is a rectangle $ABCD$ with $|AB| = a$, $|BC| = b$ and $|VA| = |VB| = |VC| = |VD| = c$. Determine the area of the intersection of the pyramid and the plane parallel to the edge VA that contains the diagonal BD.