

PROBLEMS FOR MAY

Please send your solution to
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no later than June 30, 2004. It is important that your complete mailing address and your email address appear on the front page.

Notes. The absolute value z of the number z is defined by

$$|z| = \begin{cases} z, & \text{if } z \geq 0 \\ -z, & \text{if } z < 0. \end{cases}$$

311. Given a square with a side length 1, let P be a point in the plane such that the sum of the distances from P to the sides of the square (or their extensions) is equal to 4. Determine the set of all such points P .
312. Given ten arbitrary natural numbers. Consider the sum, the product, and the absolute value of the difference calculated for any two of these numbers. At most how many of all these calculated numbers are odd?
313. The three medians of the triangle ABC partition it into six triangles. Given that three of these triangles have equal perimeters, prove that the triangle ABC is equilateral.
314. For the real numbers a , b and c , it is known that

$$\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ac} = 1,$$

and

$$a + b + c = 1.$$

Find the value of the expression

$$M = \frac{1}{1 + a + ab} + \frac{1}{1 + b + bc} + \frac{1}{1 + c + ca}.$$

315. The natural numbers 3945, 4686 and 5598 have the same remainder when divided by a natural number x . What is the sum of the number x and this remainder?
316. Solve the equation

$$|x^2 - 3x + 2| + |x^2 + 2x - 3| = 11.$$

317. Let $P(x)$ be the polynomial

$$P(x) = x^{15} - 2004x^{14} + 2004x^{13} - \dots - 2004x^2 + 2004x,$$

Calculate $P(2003)$.