PROBLEMS FOR JANUARY

Please send your solution to

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no later than February 28, 2007. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

479. Let x, y, z be positive integer for which
$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$
and the greatest common divisor of x and z is 1. Prove that $x + y$, $x - z$ and $y - z$ are all perfect squares. Give two examples of triples (x, y, z) that satisfy these conditions.

480. Let a and b be positive real numbers for which $60^a = 3$ and $60^b = 5$. Without the use of a calculator or of logarithms, determine the value of
$$12^{\frac{1-a-b}{2(1-b)}} .$$

481. In a certain town of population $2n + 1$, one knows those to whom one is known. For any set A of n citizens, there is some person among the other $n + 1$ who knows everyone on A. Show that some citizen of the town knows all the others.

[This problem was published as #11262 in the *American Mathematical Monthly* (113:10 (December, 2006), 940. Solvers of this problem should send their solutions to Prof. Barbeau and are invited to submit their solutions to the problems editor for the *Monthly*: Prof. Doug Hensley, Monthly Problems, Department of Mathematics, Texas A & M University, 3368 TAMU, College Station, TX 77843-3368, USA. A pdf file of the solution may be sent to *monthlyproblems@math.tamu.edu*.]

482. A trapezoid whose parallel sides have the lengths a and b is partitioned into two trapezoids of equal area by a line segment of length c parallel to these sides. Determine c as a function of a and b.

483. Let A and B be two points on the circumference of a circle, and E be the midpoint of arc AB (either arc will do). Let P be any point on the minor arc EB and N the foot of the perpendicular from E to AP. Prove that $AN = NP + PB$.

484. ABC is a triangle with $\angle A = 40°$ and $\angle B = 60°$. Let D and E be respective points of AB and AC for which $\angle DCB = 70°$ and $\angle EBC = 40°$. Furthermore, let F be the point of intersection of DC and EB. Prove that $AF \perp BC$.

485. From the foot of each altitude of the triangle, perpendiculars are dropped to the other two sides. Prove that the six feet of these perpendiculars lie on a circle.