PROBLEMS FOR JULY

Please send your solution to

Rosu Mihai
54 Judith Crescent
Brampton, ON L6S 3J4

no later than August 31, 2007. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

Note: The notation $\log^k x$ denotes the kth power of $\log x$, namely $(\log x)^k$.

507. Prove that, if a, b, c are positive reals, then

$$\log^2 \frac{ab}{c} + \log^2 \frac{bc}{a} + \log^2 \frac{ca}{b} + \frac{3}{4} \geq \log(abc) \ .$$

508. Let a, b, c be integers exceeding 1 for which both $\log_a b + \log_b a$ and $\log_a^2 b + \log_b^2 a$ are rational. Prove that, for every positive integer n, $\log_a^n b + \log_b^n a$ is rational.

509. Let $ABCDA'B'C'D'$ be a cube where the point O is the centre of the face $ABCD$ and $|AB| = 2a$. Calculate the distance from the point B to the line of intersection of the planes $A'B'O$ and $ADD'A'$ and the distance between AB' and BD.

510. Solve the equation

$$\sqrt[3]{x^2+2} + \sqrt[3]{4x^2+3x-2} = \sqrt[3]{3x^2+x+5} + \sqrt[3]{2x^2+2x-5} \ .$$

511. Find the sum of the last 100 digits of the number

$$A = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot 2005 \cdot 2006 + 2007 \ .$$

512. Prove that

$$\binom{3n}{n} = \sum_{k=0}^{n} \binom{2n}{k} \binom{n}{k}$$

when $n \geq 1$.

513. Solve the equation

$$2^{1-2\sin^2 x} = 2 + \log_2(1 - \sin^2 x) \ .$$