PROBLEMS FOR NOVEMBER

Please send your solution to

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no later than December 30, 2007. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

521. On a 8×8 chessboard, either $+1$ or -1 is written in each square cell. Let A_k be the product of all the numbers in the kth row, and B_k the product of all the numbers in the kth column of the board $(k = 1, 2, \cdots, 8)$. Prove that the number

$$A_1 + A_2 + \cdots + A_8 + B_1 + B_2 + \cdots + B_8$$

is a multiple of 4.

522. (a) Prove that, in each scalene triangle, the angle bisector from one of its vertices is always "between" the median and the altitude from the same vertex.

(b) Find the measures of the angles of a triangle if the lengths of the median, the angle bisector and the altitude from one of its vertices are in the ratio $\sqrt{5} : \sqrt{2} : 1$.

523. Let ABC be an isosceles triangle with $AB = AC$. The segments BC and AC are used as hypotenuses to construct three right triangles BCM, BCN and ACP. Prove that, if $\angle ACP + \angle BCM + \angle BCN = 90°$, then the triangle MPN is isosceles.

524. Solve the irrational equation

$$\frac{7}{\sqrt{x^2 - 10x + 26} + \sqrt{x^2 - 10x + 29} + \sqrt{x^2 - 10x + 41}} = x^4 - 9x^3 + 16x^2 + 15x + 26 \ .$$

525. The circle inscribed in the triangle ABC divides the median from A into three segments of the same length. If the area of ABC is $6\sqrt{14}$, calculate the lengths of its sides.

526. For the non-negative numbers a, b, c, prove the inequality

$$4(a + b + c) \geq 3(a + \sqrt{ab} + \sqrt[3]{abc}) \ .$$

When does equality hold?

527. Consider the set A of the $2n-$digit natural numbers, with 1 and 2 each occurring n times as a digit, and the set B of the $n-$digit numbers all of whose digits are 1, 2, 3, 4 with the digits 1 and 2 occurring with equal frequency. Show that A and B contain the same number of elements (*i.e.*, have the same cardinality).