PROBLEMS FOR JANUARY

Please send your solution to

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no later than February 29, 2008. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

Notes. $\lfloor x \rfloor$, known as the "floor of x, is the largest integer n that does not exceed x, *i.e.*, that integer n for which $n \leq x < n+1$. The notation $[PQR]$ denotes the area of the triangle PQR. A *geometric progression* is a sequence for which the ratio of two successive terms is always the same; its nth term has the general form ar^{n-1}.

528. Let the sequence $\{x_n : n = 0, 1, 2, \cdots\}$ be defined by $x_0 = a$ and $x_1 = b$, where a and b are real numbers, and by

$$7x_n = 5x_{n-1} + 2x_{n-2}$$

for $n \geq 2$. Derive a formula for x_n as a function of a, b and n.

529. Let k, n be positive integers. Define $p_{n,1} = 1$ for all n and $p_{n,k} = 0$ for $k \geq n+1$. For $2 \leq k \leq n$, we define inductively

$$p_{n,k} = k(p_{n-1,k-1} + p_{n-1,k}) \ .$$

Prove, by mathematical induction, that

$$p_{n,k} = \sum_{r=0}^{k-1} \binom{k}{r} (-1)^r (k-r)^n \ .$$

530. Let $\{x_1, x_2, x_3, \cdots, x_n, \cdots\}$ be a sequence is distinct positive real numbers. Prove that this sequence is a geometric progression if and only if

$$\frac{x_1}{x_2} \sum_{k=1}^{n-1} \frac{x_n^2}{x_k x_{k+1}} = \frac{x_n^2 - x_1^2}{x_2^2 - x_1^2}$$

for all $n \geq 2$.

531. Show that the remainder of the polynomial

$$p(x) = x^{2007} + 2x^{2006} + 3x^{2005} + 4x^{2004} + \cdots + 2005x^3 + 2006x^2 + 2007x + 2008$$

is the same upon division by $x(x+1)$ as upon division by $x(x+1)^2$.

532. The angle bisectors BD and CE of triangle ABC meet AC and AB at D and E respectively and meet at I. If $[ABD] = [ACE]$, prove that $AI \perp ED$. is the converse true?

533. Prove that the number

$$1 + \lfloor (5 + \sqrt{17}))^{2008} \rfloor$$

is divisible by 2^{2008}.

534. Let $\{x_n : n = 1, 2, \cdots\}$ be a sequence of distinct positive integers, with $x_1 = a$. Suppose that

$$2 \sum_{k=1}^{n} \sqrt{x_i} = (n+1)\sqrt{x_n}$$

for $n \geq 2$. Determine $\sum_{k=1}^{n} x_k$.