PROBLEMS FOR JULY-AUGUST

Please send your solutions to

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no later than August 31, 2008. Electronic files can be sent to *rosumihai@yahoo.ca*. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

556. Let x, y, z be positive real numbers for which $x + y + z = 4$. Prove the inequality

$$\frac{1}{2xy + xz + yz} + \frac{1}{xy + 2xz + yz} + \frac{1}{xy + xz + 2yz} \leq \frac{1}{xyz} \ .$$

557. Suppose that the polynomial $f(x) = (1+x+x^2)^{1004}$ has the expansion $a_0+a_1x+a_2x^2+\cdots+a_{2008}x^{2008}$. Prove that $a_0 + a_2 + \cdots + a_{2008}$ is an odd integer.

558. Determine the sum

$$\sum_{m=0}^{n-1} \sum_{k=0}^{m} \binom{n}{k} \ .$$

559. Let ϵ be one of the roots of the equation $x^n = 1$, where n is a positive integer. Prove that, for any polynomial $f(x) = a_0 + a_x + \cdots + a_n x^n$ with real coefficients, the sum $\sum_{k=1}^{n} f(1/\epsilon^k)$ is real.

560. Suppose that the numbers $x_1, x_2, \cdots, x_n$ all satisfy $-1 \leq x_i \leq 1$ $(1 \leq i \leq n)$ and $x_1^3 + x_2^3 + \cdots + x_n^3 = 0$. Prove that

$$x_1 + x_2 + \cdots + x_n \leq \frac{n}{3} \ .$$

561. Solve the equation

$$\left(\frac{1}{10}\right)^{\log_{(1/4)}(\sqrt[4]{x}-1)} - 4^{\log_{10}(\sqrt[4]{x}+5)} = 6 \ ,$$

for $x \geq 1$.

562. The circles $\mathfrak{C}$ and $\mathfrak{D}$ intersect at the two points A and B. A secant through A intersects $\mathfrak{C}$ at C and $\mathfrak{D}$ at D. On the segments CD, BC, BD, consider the respective points M, N, K for which $MN \| BD$ and $MK \| BC$. On the arc BC of the circle $\mathfrak{C}$ that does not contain A, choose E so that $EN \perp BC$, and on the arc BD of the circle $\mathfrak{D}$ that does not contain A, choose F so that $FK \perp BD$. Prove that angle EMF is right.